Mobility and stability of robots on rough terrain: modeling and control

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Abstract—On rough non-cohesive terrain, mobility or stability of a mobile robot could be critical. Then, control and planning processes must be based on relevant indexes which qualify system performances or the risk of immobility or instability. Basically, the two concepts of mobility and stability could be generalized by the one of force transmission between the contact, joint and task frames. Some methods directly inspired from manipulation or grasping applications will be used here for characterizing the obstacle clearance of articulated mobile robots or their stability on uneven ground surface. These measures are also used for motion optimization of kinematically redundant robot such as a hybrid wheeled-legged robot and an articulated multi-monocycle vehicle.

Index Terms—Rover, rough terrain, stability, mobility, obstacle clearing

I. INTRODUCTION

Robotic systems use propulsion and sustained devices such as wheel, track or foot integrated into a mechanical system. Generally, they have internal mobilities which offer an adaptation capacity according to the environment conditions and to the ground geometrical complexity. Locomotion systems can be seen as an articulated mechanical systems interacting with its environment by a set of unilateral supports which are either adhesive or slipping contact. The number and the features of these supports change over time and space. The performances of these systems are largely governed by the dynamics of the interaction between the propulsion device and the environment. Furthermore, the geometry and physical characteristics of natural ground are eminently variables. The design and the control of locomotion systems look for compromising a number of performance criteria whose most important concern the velocity, the energy, the stability, the clearing, the manoeuvrability, the cost, etc...

Many models for characterization of locomotion systems exists, going from local approaches as the terramechanics to global approaches as used for biological systems. Terramechanics concerns mainly rolling systems and turned toward traction capacity evaluation as function of soil mechanical properties. For example, it defines many mobility indexes based on a dimensional analysis of the traction on soft soil and a simple characterization using a penetration resistance measure called cone index. These mobility indexes are then correlated empirically to the system parameters [1]. Terramechanics defines also a measure for traction efficiency which gives the optimal slippage ratio for a driven wheel on a loose ground and when the desired drawbar pull is important.

In opposite to terramechanics, biology and biomechanics propose global methods that analyze animal locomotion principles [2], and their extension to artificial locomotion systems such as wheels and tracks [3]. Particularly, land locomotion modes, namely walking, running, peristaltic crawling, serpentine crawling and rolling, could be compared according different criteria, especially the mechanical cost of transport defined by the work required to move an unit mass along an unit distance. Biology shows the existence of invariant in land locomotion as the adimensional Froude number \( v^2/(gl) \) where \( v \) depicts the velocity, \( g \) the gravity and \( l \) a characteristic length. This number defines the dynamic similarity of system motion when changing scale. For example, transitions from walk to run, and from trot to gallop, take place at the same Froude number, as well for a small cat as for a big rhinoceros.

Robotics introduces many new measures for vehicle performance evaluation as kinematic mobility of wheeled robots [4], their dynamic manoeuvrability in presence of slippage [5] and their static and dynamic stability margins.

This paper deals with methods used for mobility and stability characterization of wheeled articulated robots on natural irregular terrain. First, we give the structure of their mechanical models and their interaction models with natural soils. Section (3) analyzes the velocity and force transmissions of locomotion system and proposes an extension of usual manipulation measures applied to the obstacle clearing evaluation of an articulated wheeled robot. Section (4) discusses the problem of modeling and control the mobility and the stability of locomotion systems with high degrees of freedom. A kinematic decoupling-based analysis is proposed and applied to our hybrid wheeled and legged robot. Finally, in the last section (5), we present briefly some results on performance evaluation of locomotion modes of this robot as function of geometrical and physical ground parameters.
II. GENERAL FORM OF ARTICULATED ROVER MODEL

A. From manipulation to locomotion

Model formulation of locomotion systems can take advantage from methods which characterize grasping and manipulating parallel systems. On the one hand, manipulation with a robotic hand consists in grasping, manipulating an object without releasing it. The grasping manipulation is defined by equations of movement, properties of grasping and by contact models between prehension devices (e.g. robotic fingers) and the object. On the other hand, the locomotion of a mobile robot consists in moving and ensuring stability of a robotic platform in a highly uncertain environment. This locomotion is also defined by motion equations and by contact models between locomotion devices (e.g. wheels, legs, hybrid, ...) and the ground. Grasping and locomotion issues are both based on an interaction with an object (notwithstanding the size of the object) which is stated by different models of contact (figure 1). From a pure kinematics point a view, an articulated system with unilateral contacts with ideal rolling condition can be replaced with another one which eliminate the rolling bodies and by using bilateral conventional sliding and revolute joint. For example, the simple vehicle given in figure (2) could be modelized by an usual mechanism where the velocity of the sliding joint replace the rolling velocity of the wheel i.e. \( \dot{\lambda}_i = R\dot{\theta}_i \), \( R \) is the wheel radius.

B. Kineto-static formulation

Basically, kineto-static equations can be seen as a set of linear algebraic transformation between task, joint and contact parameters (4). Let’s define

- \( \dot{\theta}, \tau \) the vectors of generalized velocity and torque of joints,
- \( \dot{x}, w \) are respectively the twist of the reference body \( S_P \) with respect to ground and the wrench transmitted by contacts on \( S_P \).

Those parameters are related linearly by two transformation matrices \( J \) and \( G \). \( J \) is the Jacobian matrix which contains jacobian of each joint expressed in contact points and with respect to the reference body (\( S_P \)). \( G \) can be defined by two ways : (1) as the locomotion matrix, its columns are the geometrical wrench of interaction constraints at each contact, or (2) as the grasp matrix of the ground body \( S_0 \). The general kineto-static model takes the following form :

\[
\begin{align*}
\dot{v} &= G^T \dot{x} = J \dot{\theta} & \text{no slip} (1) \\
\dot{v}_s &= G^T \dot{x} - J \dot{\theta} & \text{slip} (2) \\
\begin{cases}
Gf &= w \\
J^T \Gamma &= \tau 
\end{cases} (3)
\end{align*}
\]

The differential kinematic model plays a fundamental role for robot performance analysis (mobility, input/output velocity transmission, singularities analysis, traction transmission, etc ...), for odometry-based localization and for path tracking control. The method for deriving the input/output velocity relationship, which is widely used for wheeled mobile robots, consists on introducing geometrical transformations between the moving bodies and their derivative in order to obtain velocity equations by assuming ideal rolling conditions as closed-loop constraints [6]. Systematic formulations have been developed for various combinations of driving and steering wheels [7] [8]. Re-
alistic sliding models in the wheel/ground interaction had been also introduced for developing more complete models [9] [10] [5].

For eliminating the passive joint parameters in the differential kinematic equations and for obtaining the closed form of the input/output velocity relationship, we can use screw theory and reciprocal screws applied to constrained asymmetrical parallel system that provides directly the input/output equations of instantaneous velocities. Moreover, the approach gives a better geometrical insight into the problem of singularities and more generally into motion and force transmission characteristics of the system [11].

C. Dynamic formulation

Lagrange equations associated with the set of parameters \( q = (x, \theta) \) yield to the following general form of motion equation

\[
\begin{bmatrix}
M \\
\tau \\
\dot{f}
\end{bmatrix} + C + g = \begin{bmatrix}
0 \\
0 \\
\dot{f}
\end{bmatrix}
\]

\[
\begin{bmatrix}
G \\
-J
\end{bmatrix} f = \begin{bmatrix}
0 \\
0 \\
\dot{f}
\end{bmatrix}
\]

(4)

where

- \( M(x, \theta) \) is the mass matrix of the system,
- \( C(x, \theta, \dot{x}, \dot{\theta}) \) is the vector of generalized centrifuge and Coriolis forces,
- \( g \) is the generalized force due to gravity and could be computed by \( g = \begin{bmatrix} g_x = \frac{\partial U}{\partial x} = w \\ g_{\theta} = \frac{\partial U}{\partial \theta} \end{bmatrix} \) with \( U \) the total potential energy.
- \( \tau \) is the vector of joint torques.
- \( f = \lambda \) are contact forces and correspond also to Lagrange multiplier associated with relationships between parameters \((x, \theta)\) of the equation (1).

If the contacts are considered to be ideal (rolling without slipping), this last motion equation associated with the differential kinematic model (equation 1) establish the called algebraic-differential dynamic model. In the opposite case, robot-interaction models must be added to differential equations of motion, providing a determinated dynamics of the mechanical system.

D. Robot-ground interaction model

Robot-ground interaction is of high importance in land locomotion. Efficient vehicle navigation must be based on a realistic model that characterized force and velocity transmission through this contact. In first approximation, the contact can be modeled by an ideal rolling contact without slipping and a Coulomb friction law. This is a first order model which is commonly used for grasping and for locomotion analysis. However, this model could be not sufficient when the friction and the stiffness of the contact are slight. For example, on loose soil or non-cohesive granular sandy ground the optimization of the traction of a rolling vehicle has to consider the wheel slippage model. For off-road as well as for on-road vehicles, the contact model expresses the force and moment components as function of contact geometry, relative displacement parameter and their time-derivative. For the two typical simple cases of a rigid wheel on soft terrain and a flexible wheel on a hard surface, the interaction models could have an analytical form. However, for the general complex case where both the wheel and the ground are non-rigid, the models are more complex and have implicit form. We must notice, that this last case is a quite particular case because it occurs only when the stiffness of both the wheel and the ground have comparable values, otherwise we could assume one of the two typical simple cases. Mostly, the model can be written as

\[
f = f(q, \dot{q}, \ldots)
\]

(5)

\( f \) has non-linear structure and depend on contact geometry, wheel-ground material stiffness and wheel-ground friction parameters. Generally, tangential forces (called longitudinal and lateral forces) are expressed as function of two sliding parameters : the slippage ratio \( s \) and the slipping angle \( \alpha \) which are directly related to time-derivative parameters \( \dot{q} \). We notice that tangential forces are frictional dissipative forces and act against slippage velocities. Contrary to Coulomb friction model, creating tractive, braking or lateral friction force without slippage is not compatible with these models.

III. FORCE AND VELOCITY TRANSMISSION ANALYSIS

By using reciprocal screw method for elimination of passive time-derivative parameters, we obtain a relevant geometrical insight into the problem of singularities and more generally into motion and force transmission characteristics of the system. We show that the input/output differential kinematic model can be given analytically quite easily by using reciprocal screws technique [11].

\[
B\dot{\alpha} = D\dot{x}
\]

(6)

Moreover, the elements of this model have a direct physical meaningful which could help for performance analysis and optimization design. The lines of matrix \( D \) correspond to the wrench (generalized force) created by active joint and transmitted to the platform.

The RobuROC6 (figure 5) kinematics can be considered as a series of 3 monocycle modules linked together by two orthogonal revolute passive joints allowing roll and pitch motion of each module. Each monocycle module is steered and driven by two actuated conventional wheels on which a lateral slippage may occur. The rear and the front modules are symmetrically arranged about the central module. The two revolute joints along the pitch axis are coupled asymmetrically by means of 4 hydraulic actuator with interconnected chambers. This kinematic design permits to transform RobuROC6 in a 4-wheels configuration (figure III). This system can be seen as a parallel system with three limbs. Here, the central module can be defined as the moving platform and is connected to the ground by a differential steering system as well as
by the rear and the front modules via two passive revolute joints which can be seen as 2 others "limbs" connecting the reference body (platform) to the ground.

It is interesting to be able to compare the traction capabilities for different contact conditions. Hence, the set of forces and moments realizable by \( \tau \) such that \( \| \tau \| \leq 1 \) form a unit ball and if we take a particular interest in force transmission, \( D_f \) a representative measure \( \sigma \) of the traction derived from the image of this unit ball can be derived:

\[
\sigma = [\det(D_f^T D_f)]^{1/2}
\]

To show the evolution of the force transmission index \( \sigma \), we have studied the case of a step clearing for Roburoc6. Obstacle clearing and more generally traction, are highly dependent on friction forces which are directly proportional to their respective normal force \( f_n \). We can take into account normal forces by including a weighting matrix \( P \) which considers their distribution over contacts, and then compute the singular values of the weighted force transmission matrix \( D_f^T P D_f \).

Figure (7) illustrates the traction ellipsoids (with solid line) and the weighted one (with dashed line) during a step clearing. This represents force transmission factors from the joint torques frame (solid line), and from the tangential contact force frame (dashed line), to the force task frame. This measure could be improved by projecting in the task frame the contact unilaterality condition. More generally, the problem of obstacle clearing of off-road robots needs to be formulated theoretically, may be by inspiring from the widely studied grasping issue. However, we notice some experimental and numerical contributions to this problem in [12] and [13].

IV. MOBILITY VS STABILITY

On irregular 3D surface, robot motion control has to consider simultaneously the path tracking task and stability constraint. For a conventional system with passive suspension, the stability constraint will give condition on the velocity displacement along the prescribed path. However, for a high mobility articulated system as the wheeled-legged robot (fig.8), leg’s mobilities act simultaneously on load distribution and traction forces. In the same time, the rolling traction force depends strongly on the normal component of contact force. Assuming punctual and frictional contacts, this coupling problem can be formulated in the same way as for grasp problem. The contact continuity and adhesion conditions between the propulsion device and the ground can be used for defining a capacity measure of the support system to resist to external perturbations (gravitational and inertial). By using an appropriate weighting of contact forces which takes into account both uncommendable normal forces and actuator limits, one measure can be the radius of the hypersphere centered at the origin and inscribed in the wrench space. This measure is equivalent to the distance from the origin to the closest facet of the convex hull defined by the contact constraints. Some variants of this measure are proposed in [14] [15] in particular to make this measure invariant with respect to the origin point [16]. Analogue methods based on polyhedral convex cone approximation can be found in [17] [18] [19].

A. Contact force distribution

For a given task wrench, expressed by the magnitude \( \alpha \), and which ensure the vehicle motion, the contact force distribution can be based on the maximization of the objective function

\[
\Phi(f, \alpha) = \alpha - \varsigma \log \det P^{-1}
\]

where the second term \( \log \det P^{-1} \) allows to keep the contact forces away from the friction cone boundaries defined by the matrix \( P \) based on a Linear Matrix Inequality Formulation (LMI) [20]. The function \( \log \det P^{-1} \) gives a robust solution as it decreases to \( -\infty \), if any friction force approaches the cone boundary. The factor \( \varsigma \) allows a weighting of both contributions to the
with small width like quad, the stability along the roll axis is seriously threatened when turning at high speed, the lateral transfer of the load is used to measure the instability degree by [27].

We apply this tipover measure to the control of the stability of our hybrid wheeled-legged robot Hylos (fig.8). This robot has 16 degrees of freedom and is composed by 4 legs, each one has two revolute parallel axes and is ended by an active and steered wheel. This robot can perform numerous locomotion modes as pure rolling, rolling with reconfiguration, walking, peristalsis etc... The most interesting is the second one where leg’s motions are used to control stability when the robot is rolling over a 3D surface. We show that when inertial forces are negligible with respect to gravitational ones, a constant posture - defined by zero pitch and roll angles, and a nominal ground clearance - is sufficient to keep the stability margin close its maximal value [28] [29]. This control could be interesting for example for visual servoing applications where the image stability is required. However, this criteria is inefficient regarding the energy consumption as the robot uses continuously its internal mobilities to maintain constant its configuration. An alternative method represents the instability as a forbidden area defined in the configuration space to be avoided as an obstacle by applying a repulsive potential artificial potential field. This method is very useful and can fuse very easily many other functionalities as reaching a goal position, avoiding ground and obstacle collisions, avoiding joint stops etc... Figure (9) plots the stability margins with and without using internal mobilities as function of time during traversing a strongly irregular terrain displayed on figure (10). We notice that as for the well-known FIRAS function [30], the repulsive potential function creates repulsion only when the stability margin is smaller than a limit value.

Path tracking and stability control of Hylos is based on a kinematic decoupling on the operational task space of path parameters (usual horizontal position and yaw angle)
and posture parameters. This last is composed by pitch and roll angles, platform vertical height and 4 others internal parameters defined by the robot wheelbasis, since this robot exhibits high redundancy in its sagittal plane. By using the time-derivative of tracking errors of path and posture parameters with respect to the prescribed ones and using the differential kinematic model of the system, the active joint’s rates are then computed and used for actuator control [31].

V. TOWARDS A MULTI-MODES LOCOMOTION

As said before, Hylos robot is a highly redundant system and can perform several locomotion modes as function of conditions of its close environment. This adaptation of locomotion parameters can be based on models which define performance measures of locomotion modes for a given physical and geometrical ground parameters. This adaptation operates as high level control (motion planner) and requires an on-line environment sensing and characterization (if there is no a priori knowledge of the environment) (fig.11).

We are interested for the Hylos robot to 3 particular locomotion modes (or generalized gait):

- Mode (1) pure rolling : continuous rolling without walking,
- Mode (2) rolling with reconfiguration : continuous rolling with walking,
- Mode (3) peristalsis : a symmetrical walking gait with continuous contact and discontinuous rolling.

Three performance criteria are evaluated for these locomotion modes:

- stability : stability limit on slopes with different yaw configurations,
- gradability : traction limit on slopes with different yaw configurations and for different mechanical terrain parameters,
- energy : energy consumption per unit of traveled distance on slopes for different mechanical terrain parameters.

Actuator torque limits and joint stops are considered in this evaluation. We use basic concepts of terramechanics for soil characterization and for modeling wheel-ground forces. For example, figure (12) represents gradability limits on a cohesive slopping terrain as function of the slope angle $\eta$ and the yaw angle $\theta$ (with respect to the slope direction). We can notice on these polar curves that (1) the behavior is not isotropic (2) it is not symmetric with respect to $\theta = 90$ deg as rolling resistances participate to vehicle braking when it goes down, (3) slope clearing could be increased with slant paths for the three modes, (4) slope clearing is improved by using reconfiguration, probably because the stability is better, (5) peristalsis exhibits the higher clearing capacity as it minimizes the rolling resistance, etc ...

We give in figure (13) the energy consumption per unit of traveled distance on a sandy soil as function of the slope angle $\eta$. As said previously, peristalsis (mode 3) provides as from a certain slope value the best efficiency according the energy criterion while the two others modes fail as rolling traction develops very high slippage on granular non-cohesive material. Lectures could refer to [32] for getting more details on these performance evaluations and analysis.

VI. CONCLUSION

We present here some models and measures that characterize mobility and stability of articulated wheeled robots on rough terrain, that can be used for locomotion analysis or for autonomous control. We exhibit numerous analogies.
Fig. 13. Energy consumption as function of the slope angle for a sandy soft terrain.

with manipulating, grasping and walking systems for which a lot of results are already available. Inertia distribution and contact stiffness could be considered to improve measures presented in this paper. In particular, the problem of obstacle clearing has to be formulated more fundamentally using optimization of contact force distribution and thus permitting the design of advanced kinematics of systems with active or passive suspensions.

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